PERCENTAGE

The word 'Percentage' or 'Per cent' means per hundred or for every one hundred or out of 100. It is a fraction whose denominator is 100. The numerator oi the fraction is known as **rate per cent.** It is usually denoted by %. Thus, x per cent means x hundredths, written as x %.

For example : $25\% = \frac{25}{100}$

Being one of the most important topics of Mathematics, it is most importantly used in comparison. It can give us an idea of performance of any organization or performance of any student in different examinations. Let us take an example. A student in his class IX secures 300 marks out of 500 and in his class X secures 252 marks out of 400. How can you compare his performance in the above mentioned classes ? In which class he has performed better ? This can be done by taking percentage of marks. This method is illustrated below :

% Marks

$$= \frac{\text{Marks Obtained}}{\text{Total Marks}} \times 100$$

Therefore, in class IX, his score is

$$\frac{300}{500} \times 100 = 60\%$$

and in class X, his score is

$$\frac{252}{400} \times 100 = 63\ \%$$

In class IX, he has secured 60% of marks and in class X he **has** secured 6 3 % of marks. Hence, his performance in **class** X is better.

In this section, first of all we shall learn following operations:

1. **To convert a given percentage into a fraction:**

Step I: Divide it (value or term) by 100 or multiply it by $\frac{1}{100}$

Step II: Remove the sign of per cent.

Example : Express 15 % as a fraction.

Sol. : $15\% = \frac{15}{100}$ or $\frac{3}{20}$

2. **To convert fraction into percentage** : For converting a given fraction into a percentage multiply it by 100 and put the sign of percentage (%) after it.

$$\boxed{\text{Value in \%} = (\text{Fraction} \times 100)\%}$$

Example: Express $\frac{5}{8}$ at rate percent .

Sol. : $\frac{5}{8} = \left(\frac{5}{8} \times 100\right)\% = \frac{125}{2}\% = 62\frac{1}{2}\%$

3. **To find percent (%) of a given number:**

Let us suppose that we have to find x % of a given quantity y, i.e., x % of y. It is given- by $\frac{xy}{100}$

More clearly, x % of y

$$= \frac{x}{100} \times y = \frac{xy}{100}$$

If we represent $\frac{xy}{100} = z$, then

xy = z x 100

or, $x = \frac{z}{y} \times 100$ or, $y = \frac{z}{x} \times 100$

The above rules are very useful. Let's take an example to illustrate it.

Example: 50% of 200 = ?

Sol.: 50% of 200

$$= 50 \times \frac{1}{100} \times 200 = 100$$

Example : What percentage of 200 is 100 ?

Sol.: You have to find x (Percentage) y = 200, z = 100

∴ x

$$= \frac{z}{y} \times 100 = \frac{100}{200} \times 100 = 50\%$$

Example : 50 % of which quantity is 100 ?

Sol. : x = 50% y = ?, z = 100.

$$\therefore\ y = \frac{z}{x} \times 100 = \frac{100}{50} \times 100 = 200$$

4. **To convert a given quantity as a percentage of another given quantity** : For converting one given quantity, say, x as a percentage of another'given quantity, say, y, we find,

$$\frac{x}{y} \times 100\%$$

Note : Both quantities should be in same unit otherwise be converted into the same unit.

Example : How many per cent is 15 cm of 1 metre?.

Sol.:

$$\frac{15\text{ cm}}{1\text{ m}} \times 100 = \frac{15\text{ cm}}{100\text{ cm}} \times 100 = 15\ \%$$

Note : Percentage is never expressed in any unit.

Ex. If a student gets 34 marks out of 50, what is the percentage of his obtained marks ?

Solution : Percentage of obtained marks

$$= \frac{34}{50} \times 100 = 68\%$$

5. **To convert percentage into decimals**: Given value of percentage i.e. x of x% is divided by hundred to replace 100 and take result in decimal.

For example : 25%

$$= \frac{25}{100} = 0.25$$

$$\Rightarrow 36\% = \frac{36}{100} = 0.36$$

Quicker Approach Place decimal at two places from extreme right if x (x %) is a positive integer.

For example : 25% = 0.25

33% = 0.33

In case of the number x (x%) being a decimal fraction shift decimal at two places left. Annex zero(s) if needful Remember, zero is annexed to the left of given digit.

For example : 2.5% = 0.025

3.5 % = 0.035

6. lo convert decimal into percentage: In this case, the above mentioned process is reversed, i.e., shift the decimal at two places right and annex the sign of per cent (%). If necessary, annex zero.

For example : $0.35 = (0.35 \times 100)\ \% = 35\%$

$0.8 = (0.8 \times 100)\ \% = 80\%$

Note : Annex zero to the right as 0.8 = 80 %.

7. To convert fraction into % equivalent: For converting a fraction into %, we simply multiply that fraction by 100.

i.e., $\frac{a}{b} = \left(\frac{a}{b} \times 100\right)\%$

Examples :

$$\frac{3}{4} = \left(\frac{3}{4} \times 100\right)\% = 75\%$$

$$\frac{6}{5} = \left(\frac{6}{5} \times 100\right)\% = 120\%$$

Fractional equivalents of important percentages are given below. Your task is to learn these equivalents by heart:

Percentage	Fractional Equivalent	Percentage	Fractional Equivalent
100%	1	$87\frac{1}{2}\%$	$\frac{7}{8}$
80%	$\frac{4}{5}$	$66\frac{2}{3}\%$	$\frac{2}{3}$
75%	$\frac{3}{4}$	$62\frac{1}{2}\%$	$\frac{5}{8}$
60%	$\frac{3}{5}$	$37\frac{1}{2}\%$	$\frac{3}{8}$
50%	$\frac{1}{2}$	$12\frac{1}{2}\%$	$\frac{1}{8}$
40%	$\frac{2}{5}$	$16\frac{2}{3}\%$	$\frac{1}{6}$
35%	$\frac{7}{20}$	$33\frac{1}{3}\%$	$\frac{1}{3}$
30%	$\frac{3}{10}$	$83\frac{1}{3}\%$	$\frac{5}{6}$
25%	$\frac{1}{4}$	$11\frac{1}{9}\%$	$\frac{1}{9}$
20%	$\frac{1}{5}$	$22\frac{2}{9}\%$	$\frac{2}{9}$
15%	$\frac{3}{20}$	$9\frac{1}{11}\%$	$\frac{1}{11}$
5%	$\frac{1}{20}$	$14\frac{2}{7}\%$	$\frac{1}{7}$

Apart from the above types following are the types of questions which are generally asked in the examinations.

TYPE - I

Ex. **1.** In an election-a candidate secures 35% votes and is defeated by the other candidate by a margin of 450 votes. Find the total number of votes.

Sol.: Let total votes = **x**

Loser gets = 35% of **x**

.-. Winner gets = (100 - 35) % of x = 65% of x

Difference = (65 - 35) % of x = 30% of x According to condition

30% of x = 450

$$\therefore x = \frac{450}{30\%} = \frac{450}{30} \times 100 = 1500$$

Quicker Approach

$$\text{Difference} = (65 - 35) = 30\% = 450$$

$$\therefore 100\% = \frac{450}{30} \times 100 = 1500$$

TYPE - II

Ex.2. A candidate secured 360 marks in the examination. His percentage is 60%. Find the maximum marks.

Sol. Let total marks = x

$\therefore$ 60% of x = 360

$$\therefore x = \frac{360}{60} \times 100 = 600$$

Ex.3. A candidate secured 20% marks in an examination and failed by 10 marks. Another candidate secured 42% marks and got 1 mark more than the marks required to pass the examination. Determine the maximum number of marks and the percentage necessary to pass the examination.

Sol. Difference in percentages of marks of both the students = 42% - 20% = 22%

Difference in marks obtained by both the students = 10 + 1 = 11

$\therefore$ 22% = 11

$$\therefore 100\% = \frac{11}{22} \times 100 = 50$$

$\therefore$ Maximum marks = 50

Marks obtained by the successful candidate

$$= 50 \times \frac{42}{100} = 21$$

Sine? he gets 1 mark more than the bare pass marks .

$\therefore$ Bare pass marks = 21 – 1 = 20

$\therefore$ Percentage necessary to pass the examination

$$= \frac{20}{50} \times 100\% = 40\%$$

Ex.4. In an examination , 80% of the students passed in English, 85% in Mathematics and 75% in both English and Mathematics. If 40 failed in both the subjects, find the total no. of students.

Prior to its solution, let us discuss another approach :

Let n (A)= No. of members in group or set A. n (B) = No. of members in group B. N (A n B) = No. of members in both groups A and B

N (A u B) = No. of members in either group A or B or both

Then, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Now, we shall try to solve the above question by this method.

Sol.: Let the total number of students be x.

Number of students passed in one or both subjects is given by

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Here $n(A) = 80\%$ of x

$n(B) = 85\%$ of x

$n(A \cap B) = 75\%$ of x

$\therefore n(A \cup B)$

$$= \frac{80}{100}x + \frac{85}{100}x - \frac{75}{100}x = \frac{90x}{100} = \frac{9}{10}x$$

$$\text{Failed in both subjects} = x - \frac{9x}{10} = \frac{x}{10}$$

$$\therefore \frac{x}{10} = 40 \Rightarrow \therefore x = 400$$

TYPE-III

An important result: If two values are respectively x% and y% more than a third value, then the first is

$\frac{100 + x}{100 + y} \times 100\%$ of the second and the second is

$\frac{100 + y}{100 + x} \times 100\%$ of the first.

Ex. 5 : Two numbers are greater than the third number by 25% and 20% respectively. What per cent of first number is the second number?

Sol.: Suppose'third number = 100

.-. First number = 100 + 25 = 125

Second number = 100 + 20 = 120

Now, if first number is 125, then second number = 120

.-. If first number is 100, then second number

$$= \frac{120}{125} \times 100 = 96$$

Hence, second number is 96% of the first number.

Corollary : If A is x% of C and B is y% of C,

$$\text{Then, } A = \frac{x}{y} \times 100\% \text{ of } B$$

$$B = \frac{y}{x} \times 100\% \text{ of } A$$

(Continued from page 97)

TYPE-IV

If a quantity A is increased by x%, then to restore it to its original value, it must be decreased by

$$\% \text{ decrease} = \frac{x}{100 + x} \times 100\%$$

Ex **6.** If a quantity is increased by 25%, by what percent the quantity thus obtained must be reduced in order to restore to its original value ?

Sol.: Required % decrease

$$= \frac{x}{100+x} \times 100\%$$

Here, $x = 25$

.-.Required % decrease

$$= \frac{25}{100+25} \times 100\% = \frac{25}{125} \times 100\% = 20\%$$

Corollary : Similarly, if a quantity is decreased by x%, then to restore it to its original value it must now be increased by,

$$\% \text{ increase } = \frac{x}{100-x} \times 100\%$$

TYPE-V

Two Important Results

(i) If the price of a commodity increases by R% then reduction in consumption, not to increase the expenditure

is $\left\{\frac{R}{100+R} \times 100\right\}\%$

i.e. % Reduction $= \frac{\text{Increase}}{100+\text{Increase}} \times 100$

(ii) If the price of a commodity decreases by R% then the increase in consumption, not to decrease the expenditure is

$$\left\{\frac{R}{100-R} \times 100\right\}\%$$

i.e. % Increase in consumption

$$= \frac{\text{Reduction}}{100-\text{Reduction}} \times 100$$

Following examples will explain these more clearly.

Ex. **7.** Price of sugar is increased by 25%. By how much percent must a householder reduce his consumption of sugar so as not to increase the expenditure?

Sol.: Let initial expenditure = Rs. 100 Increase = 25%

New price of same quantity = Rs. 125

Now, the householder must reduce his consumption of sugar in such a way that expenditure remains 100. For that he should reduce the consumption by Rs. 25.

On Rs. 125, he should reduce = Rs. 25

On Re. 1, he should reduce = Rs. $\frac{25}{125}$

∴ On Rs. 100, he should reduce

$$= \frac{25}{125} \times 100 = \frac{1}{5} \times 100 = 20\%$$

Thus, we see that % reduction in consumption

$$= \frac{25}{125} \times 100 = \frac{25}{100+25} \times 100$$

$$= \frac{\text{Increment}}{100+\text{Increment}} \times 100$$

∴ Reduction in consumption

$$= \left(\frac{R}{100+R} \times 100\right)\%$$

$$= \frac{25}{125} \times 100 = 20\%$$

Ex. **8.** Petrol prices are reduced by 10%. Find by how much % a consumer must increase his consumption of petrol so as not to decrease his expenditure ?

Sol. : In the like manner as shown above, we can show that

$$\% \text{ Increase in consumption } = \frac{\text{Reduction}}{100-\text{Reduction}} \times 100$$

$$\therefore \% \text{ Increase } = \frac{10}{100-10} \times 100 = \frac{100}{9} = 11\frac{1}{9}\%$$

TYPE-VI

SOME IMPORTANT RESULTS

(i) Let the population of a town be P now and suppose it increases at the rate of R% per annum, then :

(a) Population after n years $= P\left(1+\frac{R}{100}\right)^n$

(b) Population n years ago $= \frac{P}{\left(1+\frac{R}{100}\right)^n}$

(ii) If the present population P decreases at the rate of R% per annum, then :

(a) Population after n years $= P\left(1-\frac{R}{100}\right)^n$

(b) Population n years ago $= \frac{P}{\left(1-\frac{R}{100}\right)^n}$

(iii) If the present population P increases at the rate of R_1% for the first year and decreases at the rate of R_2% for the second year and again increases at the rate of R₃% for the third year, then :

(a) Population after 3 years

$$= P\left(1+\frac{R_1}{100}\right)\left(1-\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$$

(b) If the same is reversed from now, then

Population 3 years ago

$$= \frac{P}{\left(1+\frac{R_1}{100}\right)\left(1-\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)}$$

Ex. 9. The population of a town is 16,000 and its annual increase is 5%. What will be the population after 3 years?

Sol. : Population after 3 years $= 1600\left(1+\frac{5}{100}\right)^3$

$$= 16000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} = 2 \times 9261 = 18522$$

TYPE- VII

Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum. Then :

(i) Value of the machine after n years $= P\left(1-\frac{R}{100}\right)^n$

(ii) Value of the machine n years ago $= \frac{P}{\left(1-\frac{R}{100}\right)^n}$

Ex. 10. The value of a machine depreciates at the rate of 10% per annum. If its present value is Rs. 162000, what will be its worth after two years? What was the value of the machine 2 years ago?

Sol.: Value of the machine after 2 years

$$= \text{Rs.}\left[162000\left(1-\frac{10}{100}\right)^2\right]$$

$$= 162000 \times \frac{9}{10} \times \frac{9}{10} = \text{Rs.}131220$$

Value of the machine 2 years ago $= \text{Rs.}\frac{162000}{\left(1-\frac{10}{100}\right)^2}$

$$= \text{Rs. } 162000 \times \frac{10 \times 10}{9 \times 9} = \text{Rs. } 2,00,000$$

TYPE-VIII

Case I: First increase and then decrease but both values are the same.

Result: If the value of a quantity is first increased by x% and then decreased by the same value x%, then net change is always a decrease which is equal to x% of x i.e..

$$x \times \frac{1}{100} \times x = \frac{x^2}{100}$$

Case II: When both values are different whenever there are two"% changes A and B, then

Net percentage change $= A + B + \frac{AB}{100}$

If there is an increase or profit etc., then the % change is taken as +ve and if there is decrease or loss or discount etc.. then, it is taken as negative.

Now, this result is also helpful whenever there are three % changes, say A, B and C. In this case we have to just find out the resultant value of A and B. Let it come out to be D. Then net % change of D and C will be the final result.

In the like manner,

% Effect on revenue

$$= \frac{\text{Increase\% value} - \text{Decrease\% value} - \text{Increase\% value} \times \text{Decrease\% value}}{100}$$

If on calculation, the answer is negative, then there is a decrease in revenue.

Ex. 11. The price ofan article is increased by 20% but its sale is decreased by 10%. Find the % change in the revenue received by the seller.

Sol. : Net % change $= A + B + \frac{AB}{100}$

Here, $A = 20$

$B = -10$

$\therefore$ Net % change $= 20 - 10 + \frac{20 \times -10}{100} = 10 - 2 = 8\%$

SOME PROBLEMS TO SOLVE

1. 1.14 expressed as a per cent of 1.9 is

(1) 6% (2) 10%

(3) 60% (4) 90%

2. In an examination 80% candidates passed in English and 85% candidates passed in Mathematics. If 73% candidates passed in both these subjects, then what per cent of candidates failed in both the subjects ?

(1) 8 (2) 15

(3) 27 (4) 35

3. Half of 1 per cent, written as a decimal, is

(1) 0.2 (2) 0.02

(3) 0.005 (4) 0.05

4. If the price of a commodity is increased by 50% by what fraction must its consumption be reduced so as to keep the same expenditure on its consumption ?

(1) $\frac{1}{4}$ (2) $\frac{1}{3}$

(3) $\frac{1}{2}$ (4) $\frac{2}{3}$

5. The price of sugar is reduced by 20%. Now a person can buy 500g more sugar for Rs. 36. The original price of the sugar per kilogram was

(1) Rs. 14.40 (2) Rs. 18

(3) Rs. 15.60 4') Rs. 16.50

6. B got 20% marks less than A. What per cent marks did A got more than B ?

(1) 20 (2) 25 (3) 12 (4) 80

7. The population of a town increases every year by 4%. If its present population is 50,000, then after 2 years it will be

(1) 53,900 (2) 54,000

(3) 54,080 (4) 54,9.00

8. Rama's expenditures and savings are in the ratio 5 : 3. If her income increases by 12% and expenditure by 15%, then by how much per cent do her savings increase ?

(1) 12 (2) 7

(3) 8 (4) 13

9. The time duration of 1 hour 45 minutes is what percent of a day?

(1) 7.218 (2) 7.291

(3) 8.3 (4) 8.24

10. In an examination, 35% of the candidates failed in Mathematics and 25% in English. If 10% failed in both Mathematics and English, then how much percent passed in both the subjects ?

(1) 50 (2)55 (3) 57 (4) 60

11. Each side of a rectangular field is diminished by 40%. By how much per cent is the area of the field diminished ?

(1) 32 (2) 64 (3)25 (4) 16

12. The price of sugar rise by 25%. If a family wants to keep their expenses on sugar the same as earlier, the family will have to decrease its consumption of sugar by

(1) 25% (2) 20%

(3) 80% (4) 75%

13. The price of an article is reduced by 25% but the daily sale of the article is increased by 30%. The net eifect on the daily sale receipts is

(1) $2\frac{1}{2}$% increase (2) $2\frac{1}{2}$% decrease

(3) 2 % increase (4) 2% decrease

14. If x earns 25% more than y. What percent less does y earn than x ?

(I) 16 (2) 10 (3) 20 (4) 25

15. The cost of an article was Rs 75. The cost was first increased by 20% and later on it was reduced by 20%. The present cost of the article is

(1) Rs. 72 (2) Rs. 60

(3) Rs. 75 (4) Rs. 90

16. In an examination, 60% of the candidates passed in English and 70% of the candidates passed in Math ematics, but 20% failed in both of these subjects. If 2500 candidates passed in both the subjects, the number of candidates that appeared at the examination was

(1) 3000 (2) 3500

(3) 4000 (4) 5000

17. The ratio of the number of boys and girls in a school is 3 : 2. If 20% of the boys and 30% of the girls are scholarship holders, then the percentage ofstudents.

who do not get scholarship, is

(1) 50 (2) 72 (3) 75 (4) 76

18. A and B are two tixed points 5 cm apart and C is a point on AB such that AC is 3 cm. If the length of AC is increased by 6%, the length of CB is decreased by

(1) 6% (2) 7% (3) 8% (4) 9%

19. ihe price 01 a certain item is increased by 15%. It a consumer wants to keep his expenditure on the item the same as before, how much per cent must he reduce his consumption of that item ?

(1) 15% (2) $13\frac{1}{23}$%

(3) $16\frac{2}{3}$% (4) $10\frac{20}{23}$%

20. If the price of petrol be raised by 20%, then the percentage by which a car owner must reduce his consumption so as not to increase his expenditure on petrol is

(1) $16\frac{1}{3}$ (2) $16\frac{2}{3}$

(3) $15\frac{2}{3}$ (4) $15\frac{1}{3}$

ANSWER S

1. (3)	**2.** (1)	**3.** (3)	**4.** (2)	**5.** (2)	**6.** (2)
7. (3)	**8.** (2)	**9.** (2)	**10.** (1)	**11.** (2)	**12.** (2)
13. (2)	**14.** (3)	**15.** (1)	**16.** (4)	**17.** (4)	**18.** (4)
19. (2)	**20.** (2)				